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By substituting these in the four symmetrical functions Σx , Σx^2 , Σx^3 and Σx^4 , equations will be found determining p , q , $q'R$. Here again, in computing Σx^n the radicals all disappear; for whenever a radical is present with one sign in any root, there is always another root in which it is present with the opposite sign; thus these expressions in pairs cancel each other. Then' in deriving Σx^n , it is only necessary to preserve the terms which are free from radicals. In this way we get

$$\begin{aligned}\Sigma x &= 4p &= -a, \\ \Sigma x^2 &= 4[p^2 + q + R] &= +a^2 - 2b, \\ \Sigma x^3 &= 4[p^3 + 3p(q + R) + 3q'R] &= -a^3 + 3ab - 3c, \\ \Sigma x^4 &= 4[(p^2 + q + R)^2 + (4p^2 + 12pq' + q'^2)R + 4q(p^2 + R)] \\ &= a^4 - 4a^2b + 4ac + 2b^2 - 4d.\end{aligned}$$

From which we derive

$$\begin{aligned}p &= -\frac{a}{4}, \quad q + R = \frac{3a^2 - 8b}{16}, \quad q'R = -\frac{a^3 - 4ab + 8c}{32}. \\ R^3 - \frac{3a^2 - 8b}{16}R^2 + \frac{3a^4 - 16a^2b + 16ac + 16b^2 - 64d}{256}R \\ &\quad - \left(\frac{3a^3 - 4ab + 8c}{64}\right)^2 = 0.\end{aligned}$$

The last is a cubic equation in R , which, by the foregoing, is solvable by radicals; hence the general equation of the fourth degree is so solvable.

In forming the value of x we may attribute to R either of the three roots of this equation. When $a = 0$, the case usually treated, the equations are simpler, viz.,

$$\begin{aligned}p &= 0, \quad q + R = -\frac{1}{2}b, \quad q'R = -\frac{1}{4}c, \\ R^3 + \frac{b}{2}R^2 + \frac{b^2 - 4d}{16}R - \frac{c^2}{64} &= 0.\end{aligned}$$

If we should attempt to treat the general equation of the 5th degree in the preceding manner, we would be led to equations of higher degrees than the 5th, which must be regarded as a strong probable argument for the non-existence of an algebraic expression equivalent to the root of the general equation of this degree.

ADDITIONAL FORMULAE IN FINITE DIFFERENCES.

BY G. W. HILL.

The formulae I have given, (p. 144 of this Journal), give the values of the integrals $\int y dx$ and $\int f dx^2$ for the series of values of x , . . . $a - h$, a , $a + h$, . . .

It is generally preferable to compute them for the values, ... $a - \frac{1}{2}h$, $a + \frac{1}{2}h$, $a + \frac{3}{2}h$, ... Formulæ for this purpose can be obtained by the simple consideration, that in the scheme on p. 141, it is allowable to treat the odd orders of differences as if they were even, and the even as if they were odd.

In this way all the quantities obtained will correspond to the middle of the intervals of the former supposition. Thus, calling D^{-1} and D^{-2} in this case $D_{\frac{1}{2}}^{-1}$ and $D_{\frac{1}{2}}^{-2}$, it is evident we must have

$$D_{\frac{1}{2}}^{-1} = h \left(\int \frac{dA}{\sqrt{1 + \frac{1}{4}A^2}} \right)^{-1},$$

$$D_{\frac{1}{2}}^{-2} = \frac{h^2}{\sqrt{1 + \frac{1}{4}A^2}} \left(\int \frac{dA}{\sqrt{1 + \frac{1}{4}A^2}} \right)^{-2},$$

or, expanded in powers of A ,

$$D_{\frac{1}{2}}^{-1} = h \left(A^{-1} + \frac{1}{24} A - \frac{17}{5760} A^3 + \frac{367}{967680} A^5 - \frac{27859}{464486400} A^7 + \dots \right),$$

$$D_{\frac{1}{2}}^{-2} = h^2 \left(A^{-2} - \frac{1}{24} A^0 + \frac{17}{1920} A^2 - \frac{367}{193536} A^4 + \frac{27859}{66355200} A^6 - \dots \right).$$

The differences of the first formulæ, although they are of odd orders, are to be taken as equivalent to the simple numbers standing in the scheme on p. 141; while the differences of the second, although of even orders, are all the averages of two adjacent numbers of the same scheme

It is plain we have

$$D_{\frac{1}{2}}^{-2} = -h \frac{dD_{\frac{1}{2}}^{-1}}{dA}.$$

ON THE DISTRIBUTION OF PRIMES.

BY PROF. W. W. JOHNSON, ST. JOHN'S COLLEGE, ANNAPOLIS, MARYLAND.

I received a short time since a letter from Mr. J. W. L. Glaisher of Trinity College, Cambridge, Eng., who is engaged in reporting upon the subject of Mathematical Tables to the British Association, a short paper in which he gives a comparison of values of the integral

$$\int_x^{x'} \frac{dx}{\log x}$$

with the actual number of primes counted between x and x' . The tables